

تمرين 1 :

$$\frac{\pi}{4} - \frac{\pi}{6} = \frac{3\pi}{12} - \frac{2\pi}{12} = \frac{\pi}{12} \quad 1$$

$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right) = \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right) = \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\tan\left(\frac{\pi}{12}\right) = \frac{\sin\left(\frac{\pi}{12}\right)}{\cos\left(\frac{\pi}{12}\right)} = \frac{\frac{\sqrt{6} - \sqrt{2}}{4}}{\frac{\sqrt{6} + \sqrt{2}}{4}} = \frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}} = \frac{(\sqrt{6} - \sqrt{2})^2}{6 - 2} = \frac{8 - 2\sqrt{12}}{4} = \frac{8 - 4\sqrt{3}}{4} = 2 - \sqrt{3} \quad 2$$

يمكنك أيضا حساب $\tan\left(\frac{\pi}{12}\right)$ باستعمال الخاصية: $\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \cdot \tan b}$

تمرين 2 :

$$\sqrt{2} \cos\left(\frac{\pi}{4} - \frac{\pi}{8}\right) = \sqrt{2} \left(\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{8}\right) + \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{8}\right) \right)$$

$$\sqrt{2} \cos\left(\frac{\pi}{4} - \frac{\pi}{8}\right) = \sqrt{2} \left(\frac{\sqrt{2}}{2} \cos\left(\frac{\pi}{8}\right) + \frac{\sqrt{2}}{2} \sin\left(\frac{\pi}{8}\right) \right) = \cos\left(\frac{\pi}{8}\right) + \sin\left(\frac{\pi}{8}\right) \quad \text{لدينا:} \quad 1$$

$$\sin\left(\frac{\pi}{8}\right) + \cos\left(\frac{\pi}{8}\right) = \sqrt{2} \cos\left(\frac{\pi}{8}\right) \quad \text{منه:} \quad \sin\left(\frac{\pi}{8}\right) + \cos\left(\frac{\pi}{8}\right) = \sqrt{2} \cos\left(\frac{\pi}{8}\right) \quad \text{لدينا:}$$

$$\sin\left(\frac{\pi}{8}\right) = (\sqrt{2} - 1)\cos\left(\frac{\pi}{8}\right) \quad \text{منه:} \quad \sin\left(\frac{\pi}{8}\right) = \sqrt{2} \cos\left(\frac{\pi}{8}\right) - \cos\left(\frac{\pi}{8}\right) \quad \text{منه:}$$

$$\tan\left(\frac{\pi}{8}\right) = \sqrt{2} - 1 \quad \text{أي:} \quad \frac{\sin\left(\frac{\pi}{8}\right)}{\cos\left(\frac{\pi}{8}\right)} = \sqrt{2} - 1 \quad \text{بالتالي:} \quad 2$$

يمثل التمرين طريقة أخرى لحساب قيمة $\tan\left(\frac{\pi}{8}\right)$

$$b \in \left] \frac{\pi}{2}; \pi \right[, a \in \left[0; \frac{\pi}{2} \right[, \sin b = \frac{3}{7} , \cos a = \frac{1}{4} \quad \text{تمرين 3:}$$

$$\sin^2 a = 1 - \frac{1}{16} = \frac{15}{16} \quad \text{منه:} \quad \sin^2 a + \left(\frac{1}{4}\right)^2 = 1 \quad \text{منه:} \quad \sin^2 a + \cos^2 a = 1 \quad \text{نعلم أن:}$$

$$\sin a = \frac{\sqrt{15}}{4} \quad \text{وبما أن:} \quad a \in \left[0; \frac{\pi}{2} \right[\quad \text{فإن:} \quad \sin a > 0 \quad \text{بالتالي:}$$

$$\cos^2 b = 1 - \frac{9}{49} = \frac{40}{49} \quad \text{منه:} \quad \left(\frac{3}{7}\right) + \cos^2 b = 1 \quad \text{منه:} \quad \sin^2 b + \cos^2 b = 1 \quad \text{نعلم أن:}$$

$$\cos b = \frac{-\sqrt{40}}{7} \quad \text{وبما أن:} \quad b \in \left] \frac{\pi}{2}; \pi \right[\quad \text{فإن:} \quad \cos b < 0 \quad \text{بالتالي:}$$

$$\cos 2a = 2 \cos^2 a - 1 = 2 \times \frac{1}{16} - 1 = \frac{1}{8} - 1 = \frac{-7}{8}$$

$$\sin 2a = 2 \sin a \cos a = 2 \times \frac{\sqrt{15}}{4} \times \frac{1}{4} = \frac{\sqrt{15}}{8}$$

$$\cos 2b = 2 \cos^2 b - 1 = 2 \times \frac{40}{49} - 1 = \frac{80}{49} - 1 = \frac{31}{49}$$

$$\sin 2b = 2 \sin b \cos b = 2 \times \frac{3}{7} \times \frac{-\sqrt{40}}{7} = \frac{-6\sqrt{40}}{49}$$

تمرين 4 :

$$\begin{aligned} \cos(x+y)\cos(x-y) &= (\cos x \cos y - \sin x \sin y)((\cos x \cos y + \sin x \sin y)) \\ &= (\cos x \cos y)^2 - (\sin x \sin y)^2 = \cos^2 x \cos^2 y - \sin^2 x \sin^2 y \\ &= \cos^2 x (1 - \sin^2 y) - (1 - \cos^2 x) \sin^2 y \\ &= \cos^2 x - \cos^2 x \sin^2 y - \sin^2 y + \cos^2 x \sin^2 y \\ &= \cos^2 x - \sin^2 y \end{aligned}$$

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$$\begin{aligned} \sin(x+y)\sin(x-y) &= (\sin x \cos y + \cos x \sin y)(\sin x \cos y - \cos x \sin y) \\ &= (\sin x \cos y)^2 - (\cos x \sin y)^2 \\ &= \sin^2 x \cos^2 y - \cos^2 x \sin^2 y \\ &= (1 - \cos^2 x) \cos^2 y - \cos^2 x (1 - \cos^2 y) \\ &= \cos^2 y - \cos^2 x \cos^2 y - \cos^2 x + \cos^2 x \cos^2 y \\ &= \cos^2 y - \cos^2 x \end{aligned}$$

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تمرين 5 :

$$\begin{aligned} \sin 3x + \sin 5x &= 2 \sin\left(\frac{3x+5x}{2}\right) \cos\left(\frac{3x-5x}{2}\right) \\ \sin 3x + \sin 5x &= 2 \sin(4x) \cos(-x) \end{aligned}$$

$$\begin{aligned} \cos x + \cos 2x &= 2 \cos\left(\frac{x+2x}{2}\right) \cos\left(\frac{x-2x}{2}\right) \\ \cos x + \cos 2x &= 2 \cos\left(\frac{3x}{2}\right) \cos\left(\frac{-x}{2}\right) \end{aligned}$$

$$\begin{aligned} \sin x - \sin \frac{x}{2} &= 2 \cos\left(\frac{x+\frac{x}{2}}{2}\right) \sin\left(\frac{x-\frac{x}{2}}{2}\right) \\ \sin x - \sin \frac{x}{2} &= 2 \cos\left(\frac{3x}{4}\right) \sin\left(\frac{x}{4}\right) \end{aligned}$$

$$\begin{aligned} \cos 3x - \cos 7x &= -2 \sin\left(\frac{3x+7x}{2}\right) \sin\left(\frac{3x-7x}{2}\right) \\ \cos 3x - \cos 7x &= -2 \sin(5x) \sin(-2x) \end{aligned}$$

$$\begin{aligned} \cos a + \cos b &= 2 \cos\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right) & \cos a - \cos b &= -2 \sin\left(\frac{a+b}{2}\right) \sin\left(\frac{a-b}{2}\right) \\ \sin a + \sin b &= 2 \sin\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right) & \sin a - \sin b &= 2 \cos\left(\frac{a+b}{2}\right) \sin\left(\frac{a-b}{2}\right) \end{aligned}$$

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