

سلسلة 3	المتتاليات العددية حلول مقترحة	السنة 1 بكالوريا علوم تجريبية	
تمرين 1 : $u_0 = -1$; $u_{n+1} = \frac{9}{6-u_n}$ ، $v_n = \frac{1}{u_n-3}$			
	$v_{n+1} - v_n = \frac{1}{u_{n+1}-3} - \frac{1}{u_n-3} = \frac{1}{\frac{9}{6-u_n}-3} - \frac{1}{u_n-3} = \frac{1}{\frac{9-3(6-u_n)}{6-u_n}} - \frac{1}{u_n-3}$ <p style="text-align: right;">لدينا :</p> $= \frac{6-u_n}{9-18+3u_n} - \frac{1}{u_n-3} = \frac{6-u_n}{3(-3+u_n)} - \frac{3}{3(u_n-3)} = \frac{6-u_n-3}{3(u_n-3)} = \frac{3-u_n}{3(u_n-3)} = \frac{1}{3}$ <p>بالتالي (v_n) متتالية حسابية أساسها $r = \frac{1}{3}$ و حدها الأول $v_0 = \frac{1}{u_0-3} = \frac{1}{-4} = -\frac{1}{4}$</p>	1	
	$v_n = v_0 + r n = -\frac{1}{4} + \frac{1}{3} n = \frac{4n-3}{12}$	2	
	$u_n = \frac{1}{v_n} + 3 = \frac{12}{4n-3} + 3 = \frac{12+12n-9}{4n-3} = \frac{12n+3}{4n-3}$ <p style="text-align: right;">لدينا : $v_n = \frac{1}{u_n-3}$ منه : $u_n-3 = \frac{1}{v_n}$ منه :</p>	3	
	$S = v_0 + v_2 + \dots + v_6 = \frac{v_0 + v_6}{2} \times 7 = \frac{-\frac{1}{4} + \frac{4 \times 6 - 3}{12}}{2} \times 7 = \frac{-3 + 24 - 3}{12} \times 7 = \frac{18}{12} \times 7 = \frac{3}{4} \times 7 = \frac{21}{4}$	4	
تمرين 2 : $u_0 = 2$; $u_{n+1} = \frac{2}{5}u_n + 1$ و $v_n = u_n - \frac{5}{3}$			
	<p>لدينا : $v_{n+1} = u_{n+1} - \frac{5}{3} = \frac{2}{5}u_n + 1 - \frac{5}{3} = \frac{2}{5}u_n - \frac{2}{3} = \frac{2}{5}\left(v_n + \frac{5}{3}\right) - \frac{2}{3} = \frac{2}{5}v_n + \frac{2}{3} - \frac{2}{3} = \frac{2}{5}v_n$</p> <p>إذن (v_n) متتالية هندسية أساسها $q = \frac{2}{5}$ و حدها الأول $v_0 = u_0 - \frac{5}{3} = 2 - \frac{5}{3} = \frac{1}{3}$</p>	1	
	$u_n = v_n + \frac{5}{3} = \frac{1}{3}\left(\frac{2}{5}\right)^n + \frac{5}{3}$ ، ولدينا : $v_n = u_n - \frac{5}{3}$ ، $v_n = v_0 \times q^n = \frac{1}{3}\left(\frac{2}{5}\right)^n$	2	
	$S = v_0 + v_1 + \dots + v_{n-1} = v_0 \times \frac{1-q^n}{1-q} = \frac{1}{3} \times \frac{1-\left(\frac{2}{5}\right)^n}{1-\frac{2}{5}} = \frac{1}{3} \times \frac{1-\left(\frac{2}{5}\right)^n}{\frac{3}{5}} = \frac{5}{9}\left(1-\left(\frac{2}{5}\right)^n\right)$	3	
تمرين 3 : $u_0 = 1, u_1 = 4$; $u_{n+2} = \frac{3}{2}u_{n+1} - \frac{1}{2}u_n$ ، $v_n = u_{n+1} - u_n$			
$v_1 = u_2 - u_1$ $v_1 = \frac{11}{2} - 4 = \frac{3}{2}$	$v_0 = u_1 - u_0$ $v_0 = 4 - 1 = 3$	$u_3 = \frac{3}{2}u_2 - \frac{1}{2}u_1$ $u_3 = \frac{33}{4} - \frac{4}{2} = \frac{25}{4}$	1
	$v_{n+1} = u_{n+2} - u_{n+1} = \frac{3}{2}u_{n+1} - \frac{1}{2}u_n - u_{n+1} = \frac{1}{2}u_{n+1} - \frac{1}{2}u_n = \frac{1}{2}(u_{n+1} - u_n) = \frac{1}{2}v_n$ <p>إذن (v_n) متتالية هندسية أساسها $q = \frac{1}{2}$ و حدها الأول $v_0 = 3$</p>	2	

$$v_0 + v_1 + \dots + v_{n-1} = (u_1 - u_0) + (u_2 - u_1) + \dots + (u_n - u_{n-1}) = -u_0 + u_n = u_n - u_0 \text{ لدينا}$$

3

$$u_n - u_0 = v_0 + v_1 + \dots + v_{n-1} = v_0 \times \frac{1 - q^n}{1 - q} = 3 \frac{1 - \frac{1}{2^n}}{1 - \frac{1}{2}} = 6 \left(1 - \frac{1}{2^n}\right) \text{ لدينا حسب السؤال السابق:}$$

4

$$\text{إذن: } u_n = 6 \left(1 - \frac{1}{2^n}\right) + u_0 = 6 - \frac{6}{2^n} + 1 = 7 - \frac{6}{2^n}$$

يمكنك التحقق من صحة النتيجة بتعويض n بالقيم 0 و 1 و 2 و 3 و مقارنتها بنتائج السؤال الأول

$$\text{تمرين 4: } u_0 = 1, v_0 = 7; \quad u_{n+1} = \frac{2u_n + v_n}{3}; \quad v_{n+1} = \frac{u_n + v_n}{2}$$

$$v_2 = \frac{u_1 + v_1}{2}$$

$$u_2 = \frac{2u_1 + v_1}{3}$$

$$v_1 = \frac{u_0 + v_0}{2}$$

$$u_1 = \frac{2u_0 + v_0}{3}$$

1

$$v_2 = \frac{3+4}{2} = \frac{7}{2}$$

$$u_2 = \frac{6+4}{3} = \frac{10}{3}$$

$$v_1 = \frac{1+7}{2} = 4$$

$$u_1 = \frac{2+7}{3} = 3$$

$$w_n = u_n - v_n$$

$$w_{n+1} = u_{n+1} - v_{n+1} = \frac{2u_n + v_n}{3} - \frac{u_n + v_n}{2} = \frac{4u_n + 2v_n - 3u_n - 3v_n}{6} = \frac{u_n - v_n}{6} = \frac{w_n}{6} \text{ لدينا:}$$

أ

$$\text{إذن } (w_n)_{n \geq 0} \text{ متتالية هندسية أساسها } q = \frac{1}{6} \text{ وحدها الأول } w_0 = 1 - 7 = -6$$

2

$$w_n = w_0 q^n = -6 \times \left(\frac{1}{6}\right)^n = \frac{-1}{6^{n-1}} \text{ (ب)}$$

$$t_n = 3u_n + 2v_n$$

$$\text{لدينا: } t_{n+1} = 3u_{n+1} + 2v_{n+1} = 2u_n + v_n + u_n + v_n = 3u_n + 2v_n = t_n \text{ بالتالي: } (t_n)_{n \geq 0} \text{ متتالية ثابتة}$$

أ

$$\forall n \in \mathbb{N} \quad t_n = t_0 = 3u_0 + 2v_0 = 3 + 14 = 17 \text{ بما أن } (t_n)_{n \geq 0} \text{ متتالية ثابتة:}$$

ب

$$\begin{cases} u_n - v_n = w_n \\ 3u_n + 2v_n = t_n \end{cases} \Rightarrow \begin{cases} 2u_n - 2v_n = 2w_n \\ 3u_n - 3v_n = 3w_n \\ 3u_n + 2v_n = t_n \end{cases} \Rightarrow \begin{cases} 5u_n = 2w_n + t_n \\ 5v_n = t_n - 3w_n \end{cases} \text{ لدينا حسب ما سبق:}$$

4

$$\forall n \in \mathbb{N} \quad \begin{cases} u_n = \frac{2w_n + t_n}{5} = \frac{1}{5} \left(\frac{-2}{6^{n-1}} + 17 \right) \\ v_n = \frac{t_n - 3w_n}{5} = \frac{1}{5} \left(17 + \frac{3}{6^{n-1}} \right) \end{cases} \text{ بالتالي:}$$

يمكنك التحقق من صحة النتيجة بتعويض الصيغ المحصل عليها (يمكنك أيضا حساب بعض القيم الخاصة للتحقق من

صحة النتائج مثل u_0 و v_0)