

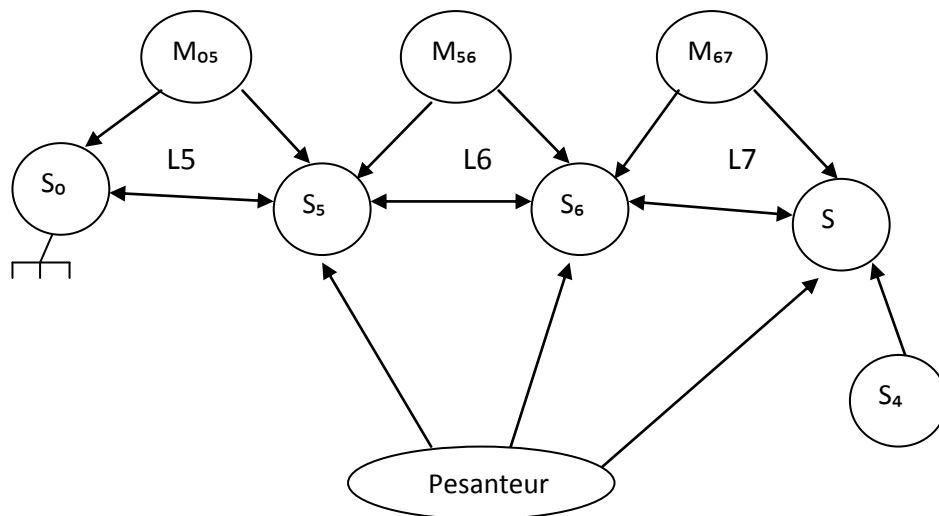
Correction de CNC TSI 2010

Génie mécanique

I. Étude du système porte utile :

1. Étude statique :

1.1.1 Schéma d'analyse :



L5 : glissière d'axe (O_5, \vec{x}_0)

L6 : glissière d'axe (O_6, \vec{x}_0)

L7 : pivot d'axe (O_7, \vec{x}_0)

1.1.2 les torseurs :

$$\left\{ \tau_{S_0 \rightarrow S_5} \right\} = \left\{ \begin{array}{l} \mathbf{O} \mid L_{05} \\ Y_{05} \mid M_{05} \\ Z_{05} \mid N_{05} \end{array} \right\}_{(O_5, B_0)} \quad \left\{ \tau_{S_5 \rightarrow S_6} \right\} = \left\{ \begin{array}{l} X_{56} \mid L_{56} \\ Y_{56} \mid M_{56} \\ \mathbf{O} \mid N_{56} \end{array} \right\}_{(O_6, B_0)}$$

$$\left\{ \tau_{S_6 \rightarrow S_7} \right\} = \left\{ \begin{array}{l} X_{67} \mid \mathbf{O} \\ Y_{67} \mid M_{67} \\ Z_{67} \mid N_{67} \end{array} \right\}_{(O_7, B_0)}$$

Rq : le dernier torseur est valable dans toute base qui contient \vec{x}_0

1.1.3

$$\left\{ \tau_{M_{67} \rightarrow S} \right\} = \left\{ \begin{array}{l} \mathbf{0} \mid \mathbf{C}_{67} \\ \mathbf{0} \mid \mathbf{0} \\ \mathbf{0} \mid \mathbf{0} \end{array} \right\}_{(O_7, B_0)} \quad \left\{ \tau_{pes \rightarrow S} \right\} = \left\{ \begin{array}{l} \mathbf{0} \mid \mathbf{0} \\ \mathbf{0} \mid \mathbf{0} \\ -m.g \mid \mathbf{0} \end{array} \right\}_{(O_7, B_0)}$$

$$\left\{ \tau_{S_6 \rightarrow S_7} \right\} = \left\{ \begin{array}{l} \mathbf{X}_{67} \mid \mathbf{0} \\ \mathbf{Y}_{67} \mid \mathbf{M}_{67} \\ \mathbf{Z}_{67} \mid \mathbf{N}_{67} \end{array} \right\}_{(O_7, B_0)} \quad \left\{ \tau_{S_4 \rightarrow S} \right\} = \left\{ \begin{array}{l} \mathbf{X}_{48} \mid \mathbf{0} \\ \mathbf{Y}_{48} \mid \mathbf{0} \\ \mathbf{Z}_{48} \mid \mathbf{0} \end{array} \right\}_{(M, B_0)}$$

- Réduction des torseurs au point O_{07} :

$$\vec{M}_{O_7} = \vec{M}_{M(S_4 \rightarrow S)} + \vec{O_7 M} \wedge \vec{R}_{S_4 \rightarrow S}; \vec{O_7 M} = \begin{pmatrix} e \\ R \\ 0 \end{pmatrix}_{(O_7, B_7)} = \begin{pmatrix} e \\ R \cos \theta_{76} \\ R \sin \theta_{76} \end{pmatrix}_{(O_7, B_0)}$$

$$\vec{M}_{O_7} = \begin{pmatrix} Z_{48} \cdot R \cos \theta_{76} - Y_{48} R \sin \theta_{76} \\ X_{48} R \sin \theta_{76} - e Z_{48} \\ e Y_{48} - X_{48} R \cos \theta_{76} \end{pmatrix}_{(O_7, B_0)} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}_{(O_7, B_0)}$$

- PFS :

$$\left\{ \tau_{\vec{S} \rightarrow S} \right\}_{O_7} = \left\{ \begin{array}{l} \vec{0} \\ \vec{0} \end{array} \right\}_{O_7} \Rightarrow \begin{cases} X_{67} + X_{48} = 0 \\ Y_{67} + Y_{48} = 0 \\ Z_{67} + Z_{48} - mg = 0 \end{cases} ; \begin{cases} C_{67} + \alpha = 0 \\ M_{67} + \beta = 0 \\ N_{67} + \gamma = 0 \end{cases}$$

- Equation du mouvement : $C_{67} + \alpha = 0$

2. Etude cinématique :

1.2.1 Torseurs cinématiques :

$$\left\{ \mathcal{G}_{S_5/S_0} \right\}_{O_5} = \left\{ \begin{array}{l} \vec{0} \\ \dot{\mu} \vec{x}_0 \end{array} \right\}_{O_5} ; \left\{ \mathcal{G}_{S_6/S_5} \right\}_{O_6} = \left\{ \begin{array}{l} \vec{0} \\ \dot{\delta} \vec{z}_0 \end{array} \right\}_{O_6} ; \left\{ \mathcal{G}_{S/S_6} \right\}_{O_7} = \left\{ \begin{array}{l} \dot{\theta}_{76} \vec{x}_0 \\ \vec{0} \end{array} \right\}_{O_7}$$

1.2.2

$$\vec{V}_{M \in S/R_0} = \left. \frac{d\vec{O_0M}}{dt} \right|_{R_0} - \left. \frac{d\vec{O_7M}}{dt} \right|_S ; \left. \frac{d\vec{O_7M}}{dt} \right|_S = \frac{d}{dt} (\vec{ex}_7 + R\vec{y}_7) \Big|_S = \vec{0}$$

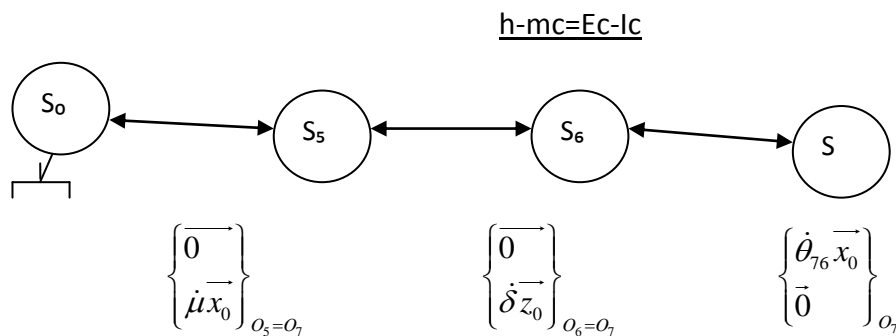
$$\left. \frac{d\vec{O_0M}}{dt} \right|_{R_0} = \left. \frac{d\vec{O_0O_5}}{dt} \right|_{R_0} + \left. \frac{d\vec{O_5O_6}}{dt} \right|_{R_0} + \left. \frac{d\vec{O_6O_7}}{dt} \right|_{R_0} + \left. \frac{d\vec{O_7M}}{dt} \right|_{R_0}$$

$$\Rightarrow \vec{V}_{M \in S/R_0} = \dot{\mu} \vec{x}_0 - R\dot{\theta}_{76} \sin \theta_{76} \vec{y}_0 + (\dot{\delta} + R\dot{\theta}_{76} \cos \theta_{76}) \vec{z}_0$$

1.2.3

$$\vec{\Gamma}_{M/R_0} = \left. \frac{d\vec{V}_{M \in S/R_0}}{dt} \right|_{R_0} \text{ Facile à calculer.}$$

3. Etude d'hyper statisme :



$$\{g_{S/0}\}_{O_7} = \left\{ \begin{matrix} \dot{\theta}_{76} \vec{x}_0 \\ \dot{\mu} \vec{x}_0 + \dot{\delta} \vec{z}_0 \end{matrix} \right\}_{O_7} \Rightarrow mc=3$$

$$H=3+0-3=0$$

(Rq et vérification : chaîne ouverte => H=0)

4. Etude de la résistance des matériaux :

$$\{\tau_{S_7 \rightarrow S_8}\}_I = -\{T\}_I \text{ Et } \vec{M}_I = \vec{M}_J + \vec{IJ} \wedge \vec{R} = \begin{pmatrix} 10^6 \\ -19900 \\ -39850 \end{pmatrix}$$

$$\text{Donc } \{\tau_{S_7 \rightarrow S_8}\}_I = \begin{pmatrix} -100 & -10^6 \\ 200 & 19900 \\ -100 & 39850 \end{pmatrix}_I$$

1.4.2

$$\{\tau_{coh}\}_G = \{T\}_G \quad \text{Et on a } \vec{M}_G = \vec{M}_J + \vec{GJ} \wedge \vec{R} = \begin{pmatrix} 10^6 \\ 100 \\ 150 \end{pmatrix} + \begin{pmatrix} L-x \\ 0 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 100 \\ -200 \\ 100 \end{pmatrix}$$

$$\text{Donc } \{\tau_{coh}\}_G = \left\{ \begin{array}{l|l} 100 & 10^6 \\ -200 & 100(1-L+x) \\ 100 & 150-200(L-x) \end{array} \right\}_G$$

1.4.3 Condition de résistance :

$$\tau_{\max} = k \cdot \frac{M_t}{I_0} \cdot V \leq \frac{\tau_e}{s} \quad \text{Avec } V = \frac{De}{2} \quad \text{et } I_0 = \frac{\pi(De^4 - Di^4)}{32} = \frac{\pi(1-0.7^4)De^4}{32}$$

Et on a : Di=0.7 De, k=1.5, s=3, Mt=10^6

AN : De=68mm ; Di=48mm

II. Étude du système porte pièce :

1) Étude cinétique :

2.1.1

$$(\vec{O}_3, \vec{y}_3) \text{ Axe de révolution } \rightarrow [I_{O_3}(\Sigma)] = \begin{pmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{pmatrix}_{(-, \vec{y}_3, -)} \quad \text{et } A = \frac{B}{2} + \int y^2 dm$$

2.1.2

$$\vec{V}_{O_3 \in \Sigma / R_0} = \left. \frac{d\vec{HO}_3}{dt} \right|_{R_0} = \dot{\lambda} \vec{y}_1 = \dot{\lambda} (\sin \theta \vec{x}_2 + \cos \theta \vec{y}_2)$$

$$\vec{\sigma}_{O_3(\Sigma/R_0)} = \overline{\overline{I_{O_3}(\Sigma)}} \cdot \overline{\overline{\Omega_{\Sigma/R_0}}} = \begin{pmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{pmatrix}_{(-, \vec{y}_3, -)} \cdot \begin{pmatrix} 0 \\ \dot{\theta}_{32} \\ \dot{\theta}_{21} \end{pmatrix}_{B_2} = \begin{pmatrix} 0 \\ B \cdot \dot{\theta}_{32} \\ A \cdot \dot{\theta}_{21} \end{pmatrix}_{B_2}$$

$$\left\{ \xi_{\Sigma/R_0} \right\}_{O_3} = \left\{ \begin{array}{l} m_{\Sigma} \cdot \vec{V}_{O_3 \in \Sigma / R_0} \\ \vec{\sigma}_{O_3(\Sigma/R_0)} \end{array} \right\}_{O_3} = \left\{ \begin{array}{l|l} \dot{\lambda} m_{\Sigma} \sin \theta & 0 \\ \dot{\lambda} m_{\Sigma} \cos \theta & B \cdot \dot{\theta}_{32} \\ 0 & A \cdot \dot{\theta}_{21} \end{array} \right\}_{(O_3, B_2)}$$

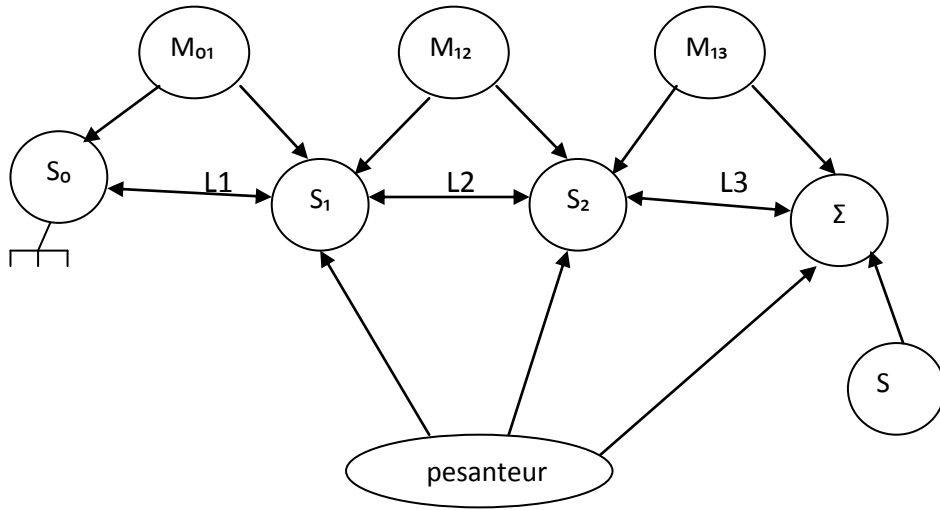
2) étude dynamique :

2.2.1

L1 : Glissière d'axe (O_1, \vec{y}_1)

L2 : Pivot d'axe (O_2, \vec{z}_2)

L3 : Pivot d'axe (O_3, \vec{y}_2)



2.2.2

$$\vec{\Gamma}_{O_3 \in \Sigma / R_0} = \left. \frac{d\vec{V}_{O_3 \in \Sigma / R_0}}{dt} \right|_{R_0} = \ddot{\lambda} \vec{y}_1 = \ddot{\lambda} (\sin \theta_{21} \vec{x}_2 + \cos \theta_{21} \vec{y}_2)$$

$$\vec{\delta}_{O_3(\Sigma/R_0)} = \left. \frac{d\vec{\sigma}_{O_3(\Sigma/R_0)}}{dt} \right|_{R_0} = \left. \frac{d\vec{\sigma}_{O_3(\Sigma/R_0)}}{dt} \right|_2 + \vec{\Omega}_{2/R_0} \wedge \vec{\sigma}_{O_3(\Sigma/R_0)} = \begin{pmatrix} -B \cdot \dot{\theta}_{32} \cdot \dot{\theta}_{21} \\ B \cdot \dot{\theta}_{32} \\ A \cdot \dot{\theta}_{21} \end{pmatrix}_{B_2}$$

$$\left\{ \mathcal{D}_{\Sigma/R_0} \right\}_{O_3} = \left\{ \begin{matrix} m_{\Sigma} \cdot \vec{\Gamma}_{O_3 \in \Sigma / R_0} \\ \vec{\delta}_{O_3(\Sigma/R_0)} \end{matrix} \right\}_{O_3} = \left\{ \begin{matrix} \ddot{\lambda} m_{\Sigma} \sin \theta & -B \cdot \dot{\theta}_{32} \cdot \dot{\theta}_{21} \\ \ddot{\lambda} m_{\Sigma} \cos \theta & B \cdot \dot{\theta}_{32} \\ 0 & A \cdot \dot{\theta}_{21} \end{matrix} \right\}_{(O_3, B_2)}$$

2.2.3 PFD

III. Etude de Fabrication : (voir cours)