

تمرين رقم (1)

[1] باستعمال التعريف للنهاية بين ما يلي :

$$\lim_{x \rightarrow -2} \left(\frac{3}{2}x - 1 \right) = -4 \quad \textcircled{2}, \quad \lim_{x \rightarrow 1} (2x + 1) = 3 \quad \textcircled{1}$$

$$\lim_{x \rightarrow 4} (\sqrt{x} - 1) = 1 \quad \textcircled{3}$$

$$f(x) = x^2 - x + 3 \quad \textcircled{1} \quad [2]$$

(1) بين أن :

$$|f(x) - 5| \leq 4|x - 2| \quad \text{فإن } |x - 2| \leq 1 \quad \text{إذا كان}$$

$$\lim_{x \rightarrow 2} f(x) = 5 \quad \text{استنتج أن} \quad \textcircled{2}$$

$$f(x) = \frac{x+1}{2x+1} \quad \text{نضع} \quad \textcircled{2}$$

$$\left| x - \frac{1}{2} \right| \leq \frac{1}{2} \Rightarrow \left| f(x) - \frac{3}{4} \right| \leq \frac{1}{2} \left| x - \frac{1}{2} \right| \quad (1) \quad \text{بين أن}$$

$$\lim_{x \rightarrow \frac{1}{2}} f(x) = \frac{3}{4} \quad \text{استنتاج أن} \quad \textcircled{2}$$

تمرين رقم (2)

أحسب النهايات التالية :

$$\lim_{x \rightarrow -2} \frac{x\sqrt{x+6} + 1}{x^2 - 1}, \quad \lim_{x \rightarrow 1} \frac{2x^2 + x - 4}{3x + 1}$$

$$\lim_{x \rightarrow 2} \frac{2x^2 + 3x - 2}{x^2 + x - 2}; \quad \lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x^2 - 4}; \quad \lim_{x \rightarrow 1} \frac{x^2 - x}{x^2 - 1}$$

$$\lim_{x \rightarrow -2} \frac{x^3 + 8}{x^2 + 3x + 2}; \quad \lim_{x \rightarrow 1} \frac{2x^2 - x}{4x^2 - 1}; \quad \lim_{x \rightarrow 1} \frac{x^3 + 1}{x^2 - 1}$$

$$\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{x^2 - 9}; \quad \lim_{x \rightarrow 2} \frac{x^2 - 4x + 4}{x^2 - 4}$$

$$\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x - 3}; \quad \lim_{x \rightarrow \frac{3}{2}} \frac{3x^2 - 5x + 2}{9x^2 - 4}$$

$$\lim_{x \rightarrow 1} \frac{x\sqrt{x} - 1}{x^2 - 1}; \quad \lim_{x \rightarrow 1} \frac{\sqrt{3x+1} - 2}{\sqrt{x} - 1}$$

تمرين رقم (3)

أحسب ما يلي :

$$\lim_{x \rightarrow -\infty} \frac{1}{2}x^3 - 4x^2 + 9; \quad \lim_{x \rightarrow +\infty} 3x^2 - 5x + 2$$

$$\lim_{x \rightarrow +\infty} \frac{2x^2 - x - 3}{(x-1)}; \quad \lim_{x \rightarrow +\infty} (2x+1)^2 - 5x^2 + 3$$

$$\lim_{x \rightarrow +\infty} \frac{x^3 - 2x}{x^2 - 2x}; \quad \lim_{x \rightarrow -\infty} \frac{3x^2 - 2x + 1}{(x-2)^2}$$

$$\lim_{x \rightarrow +\infty} \frac{x^2 - 2x}{x+1} - 2x; \quad \lim_{x \rightarrow +\infty} \frac{x+1}{2x^2 - x - 1}$$

$$\lim_{x \rightarrow -\infty} \sqrt{2-x} - x; \quad \lim_{x \rightarrow +\infty} \sqrt{x+1} + 2x$$

$$\lim_{x \rightarrow +\infty} \sqrt{2x-1} - x; \quad \lim_{x \rightarrow +\infty} \sqrt{x+1} + 2x$$

$$\lim_{x \rightarrow +\infty} \sqrt{x^2 + x} - x; \quad \lim_{x \rightarrow -\infty} \sqrt{x^2 + 1} + x$$

تمرين رقم (4)

حدد النهايات التالية :

$$\lim_{x \rightarrow -1} \frac{x^2 + 2x}{(x+1)^2}, \quad \lim_{x \rightarrow 2} \frac{x^2 - 3x}{(x-2)^2}, \quad \lim_{x \rightarrow 4} \frac{3x+5}{|x+4|}$$

$$\lim_{\substack{x \rightarrow \frac{1}{2} \\ x > \frac{1}{2}}} \frac{4x-3}{2x-1}; \quad \lim_{\substack{x \rightarrow 1 \\ x > 1}} \frac{2x+3}{x^2-1}$$

$$\lim_{\substack{x \rightarrow -3 \\ x > -3}} \frac{2x+1}{x^2-9}; \quad \lim_{\substack{x \rightarrow 2 \\ x < -2}} \frac{x(x+3)}{(x-1)(x+2)}$$

$$\lim_{\substack{x \rightarrow 0 \\ x < 0}} \left(\frac{1}{x} + \frac{x+2}{x^2} \right); \quad \therefore \quad \lim_{\substack{x \rightarrow 2 \\ x > 2}} \frac{\sqrt{x-2}}{x^2-2x}$$

$$\lim_{\substack{x \rightarrow 1 \\ x < 1}} \frac{2x^2 - x + 3}{x^2 - x}; \quad \therefore \quad \lim_{\substack{x \rightarrow -\frac{3}{2} \\ x < \frac{-3}{2}}} \frac{6x+5}{4x^2-9}$$

$$\lim_{\substack{x \rightarrow 3 \\ x > 3}} \frac{\sqrt{x-3} + x^2 - 9}{x-3}; \quad \therefore \quad \lim_{\substack{x \rightarrow 0 \\ x < 0}} \frac{x+2-2\sqrt{x+1}}{x^2}$$

تمرين رقم (5)

حدد النهايات التالية :

$$\lim_{x \rightarrow 0} \frac{x+2\sin x}{2\tan 3x-x}, \quad \lim_{x \rightarrow 0} \frac{\tan 5x}{\sin 3x}; \quad ; \quad \lim_{x \rightarrow 0} \frac{\sin 2x}{3x}$$

$$\lim_{x \rightarrow 0} \frac{1-\cos 3x}{x^2}, \quad \lim_{x \rightarrow 0} \frac{4x-\sin 2x}{x+\tan 3x}$$

$$\lim_{x \rightarrow 0} \frac{\cos 3x - \cos x}{x^2}, \quad \lim_{x \rightarrow 0} \frac{\cos x - 1}{x \sin 3x}$$

$$\lim_{x \rightarrow 0} \frac{\cos x + \cos 3x - 2}{x^2}, \quad \lim_{x \rightarrow 0} \frac{2\sin x - \sin 2x}{x^3}$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{1-\sin x}, \quad \lim_{x \rightarrow \pi} \frac{1+\cos x}{\sin x}, \quad \lim_{x \rightarrow 1} \frac{\sin(\pi x)}{x-1}$$

$$\lim_{x \rightarrow \frac{\pi}{3}} \frac{2\sin x - \sqrt{3}}{x - \frac{\pi}{3}}; \quad \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos x - \sin x}{x - \frac{\pi}{4}}$$

$$\lim_{x \rightarrow 0} \frac{1+\sin x}{1-\cos x}, \quad \lim_{x \rightarrow +\infty} \frac{\sin 3x}{2x}; \quad ; \quad \lim_{x \rightarrow +\infty} \frac{\sin x}{x}$$

$$\lim_{x \rightarrow +\infty} \frac{3-2\cos x}{x^2+1}, \quad \lim_{\substack{x \rightarrow 0 \\ x < 0}} \frac{1+\cos x}{\sin x}$$

$$\lim_{x \rightarrow 0} \frac{\cos ax - \cos bx}{x^2}, \quad \lim_{x \rightarrow +\infty} \frac{2x-3\sin x}{2\cos 3x-5x}$$