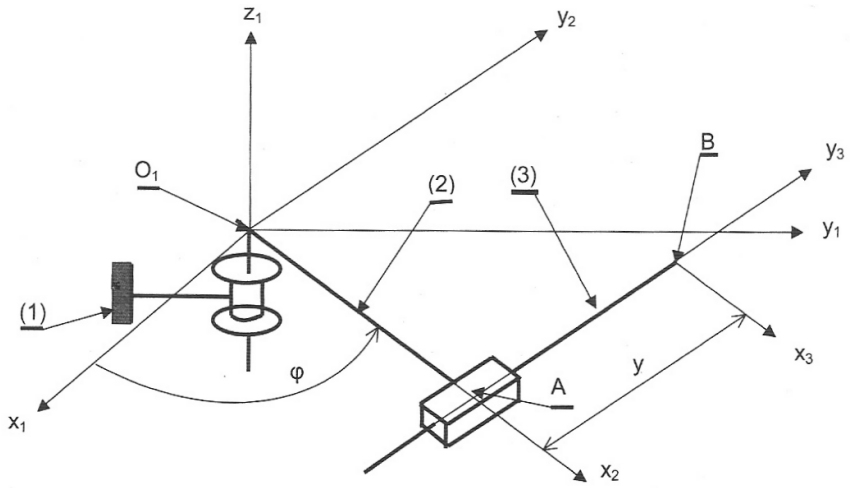
TD cinématique du solide : Vitesse d'un solide

Exercice 1 :

Robot manipulateur 2 axes



$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = a.\vec{x}_2 + y.\vec{y}_3 = a.\vec{x}_2 + y.\vec{y}_2$$

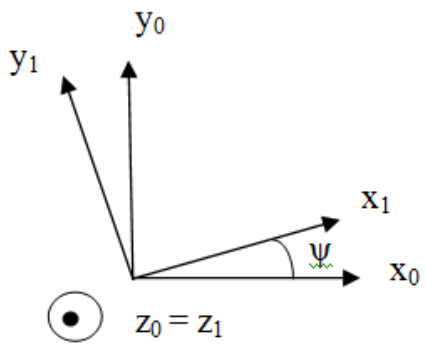
$$\vec{V}(B \in 3/1) = \left(\frac{d\overrightarrow{OB}}{dt} \right)_1$$

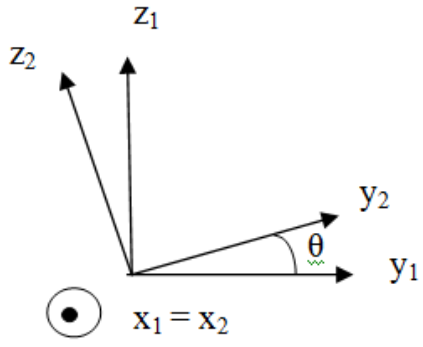
$$\vec{V}(B \in 3/1) = a.\dot{\varphi}.\vec{y}_2 + \dot{y}.\vec{y}_2 - y.\dot{\varphi}.\vec{x}_2 = (a.\dot{\varphi} + \dot{y}).\vec{y}_2 - y.\dot{\varphi}.\vec{x}_2$$

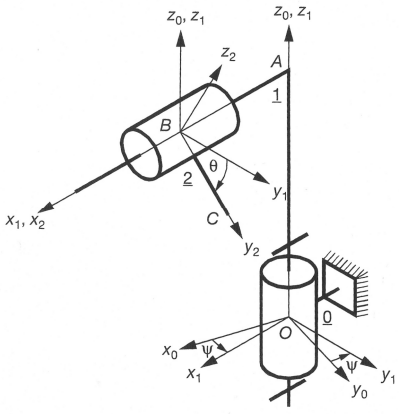
$$\vec{A}(B \in 3/1) = (a.\ddot{\varphi} + \ddot{y}).\vec{y}_2 - (a.\dot{\varphi}^2 + \dot{y}.\dot{\varphi}).\vec{x}_2 - \dot{y}.\dot{\varphi}.\vec{x}_2 - y.\ddot{\varphi}.\vec{x}_2 - y.\dot{\varphi}^2.\vec{y}_2$$

$$\vec{A}(B \in 3/1) = -(a.\dot{\varphi}^2 + 2.\dot{y}.\dot{\varphi} + y.\ddot{\varphi}).\vec{x}_2 + (a.\ddot{\varphi} + \ddot{y} - y.\dot{\varphi}^2).\vec{y}_2$$

Exercice 2 : Robot manipulateur 3 axes







$$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} = a.\vec{z}_0 + x.\vec{x}_1 + b.\vec{y}_2$$

$$\vec{V}(C \in 2/0) = \left(\frac{d\overrightarrow{OC}}{dt} \right)_0$$

$$\vec{V}(C \in 2/0) = \dot{x}.\vec{x}_1 + x.\dot{\psi}.\vec{y}_1 + b.(\frac{d\vec{y}_2}{dt})_0$$

$$\left(\frac{d\vec{y}_2}{dt} \right)_0 = \left(\frac{d\vec{y}_2}{dt} \right)_1 + \vec{\Omega}(1/0) \wedge \vec{y}_2$$

$$\left(\frac{d\vec{y}_2}{dt} \right)_0 = \dot{\theta}.\vec{z}_2 + \dot{\psi}.\vec{z}_1 \wedge (\cos\theta.\vec{y}_1 + \sin\theta.\vec{z}_1) = \dot{\theta}.\vec{z}_2 - \dot{\psi}.\cos\theta.\vec{x}_1$$

$$\vec{V}(C \in 2/0) = \dot{x}.\vec{x}_1 + x.\dot{\psi}.\vec{y}_1 + b.(\dot{\theta}.\vec{z}_2 - \dot{\psi}.\cos\theta.\vec{x}_1)$$

$$\vec{V}(C \in 2/0) = (\dot{x} - \dot{\psi}.\cos\theta).\vec{x}_1 + x.\dot{\psi}.\vec{y}_1 + b.\dot{\theta}.\vec{z}_2$$

$$\vec{A}(C \in 2/0) = (\ddot{x} - \ddot{\psi}.\cos\theta + \dot{\psi}.\dot{\theta}.\cos\theta).\vec{x}_1 + (\dot{x} - \dot{\psi}.\cos\theta).\dot{\psi}.\vec{y}_1$$

$$+ \dot{x}.\dot{\psi}.\vec{y}_1 + x.\ddot{\psi}.\vec{y}_1 - x.\dot{\psi}^2.\vec{x}_1 + b.\ddot{\theta}.\vec{z}_2 + b.\dot{\theta}.\left(\frac{d\vec{z}_2}{dt} \right)_0$$

$$\left(\frac{d\vec{z}_2}{dt} \right)_0 = \left(\frac{d\vec{z}_2}{dt} \right)_1 + \vec{\Omega}(1/0) \wedge \vec{z}_2$$

$$\left(\frac{d\vec{z}_2}{dt} \right)_0 = -\dot{\theta}.\vec{y}_1 + \dot{\psi}.\vec{z}_1 \wedge (-\sin\theta.\vec{y}_1 + \cos\theta.\vec{z}_1)$$

$$\left(\frac{d\vec{z}_2}{dt} \right)_0 = -\dot{\theta}.\vec{y}_1 + \dot{\psi}.\sin\theta.\vec{x}_1$$