Méca

A. 1) $x_{eq} = x_0 + \frac{mg}{k}$ mg en N
k en $N.m^{-1}$

2) non amorti $\Rightarrow E_m = Cte \Leftrightarrow \frac{dE_m}{dt} = 0$

$\Leftrightarrow \frac{d}{dt}\left(\frac{1}{2}m\dot{x}^2 + \frac{1}{2}k(x-x_0)^2 - mgx\right) = 0$

$\Leftrightarrow m\dot{x}\ddot{x} + k(x-x_0)\dot{x} - mg = 0$

$\Leftrightarrow m\ddot{x} + kx = kx_0 + mg$

3) $\ddot{x} + \frac{k}{m}x = \frac{k}{m}x_{eq}$ $\omega_0^2 = \frac{k}{m}$ $T_0 = 2\pi\sqrt{\frac{m}{k}}$

4) $x_H = A\cos(\omega_0 t) + B\sin(\omega_0 t)$
$x_p = x_{eq}$
$\Big\} x(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t) + x_{eq}$

CI $\dot{x}(0) = v_0 \Rightarrow B = \frac{v_0}{\omega_0}$
$x(0) = x_{eq} \Rightarrow A = 0$
$x(t) = x_{eq} + \frac{v_0}{\omega_0}\sin(\omega_0 t)$

B) 1) $m = \rho V \Rightarrow \omega_1 = \sqrt{\frac{k}{\rho V}}$

2) a) $x'_{eq} = x_0 + \frac{\rho V - \rho_e V}{k} g = x_0 + (\rho - \rho_e)\frac{Vg}{k}$

b) $\rho_e = \rho_0 + \frac{x_0 - x'_{eq}}{Vg} k$

3) a) $\rho V \ddot{x} = -k(x - x_0) + \rho V g - \rho_e V g - 6\pi\eta R\dot{x}$

b) $\ddot{x} + \frac{k}{\rho V}x + \frac{6\pi\eta R}{\rho V}\dot{x} = \frac{k}{\rho V}x'_{eq}$

c) $\omega_1^2 = \frac{k}{\rho V}$ $\frac{\omega_1}{Q} = \frac{6\pi\eta R}{\rho V} \Rightarrow Q = \frac{1}{6\pi\eta R}\sqrt{k\rho V}$

d) $\Delta < 0$ ou $Q > 1/2 \Leftrightarrow \frac{\sqrt{k\rho V}}{6\pi\eta R} > \frac{1}{2} \Leftrightarrow k > \frac{9\pi^2\eta^2R^2}{\rho V} = k_0$

e) $\Delta = \frac{36\pi^2\eta^2R^2}{\rho^2V^2} - \frac{4k}{\rho V} = \frac{4k}{\rho V}\left(\frac{k_0}{k} - 1\right)$

~~soluti~~ partie imaginaire de la solution de l'équation dif.
= pseudo pulsation $\omega_2 = \frac{\sqrt{|\Delta|}}{2} = \omega_1\sqrt{1 - \frac{k_0}{k}}$

C. 1) $\underline{x} = x_m e^{j(\omega t + \varphi)}$ $\underline{F} = F_0 e^{j\omega t}$

$\frac{d}{dt}(\dots) \Leftrightarrow j\omega \times (\dots) \rightarrow -\omega^2 \underline{x} + 2j\omega\alpha \underline{x} + \omega_0^2 \underline{x} = \frac{\underline{F}}{m}$

$\underline{x} = \frac{\underline{F}}{m(\omega_0^2 - \omega^2 + 2j\omega\alpha)}$

2) $x_m = \frac{F_0}{m\sqrt{(\omega_0^2-\omega^2)^2 + 4\omega^2\alpha^2}}$

3) $\omega t + \varphi = \underbrace{\arg(\underline{F})}_{\omega t} \underbrace{- \arctan\left(\frac{2\alpha\omega}{\omega_0^2 - \omega^2}\right)}_{\varphi}$

4) soit $f(\omega) = (\omega_0^2 - \omega^2)^2 + 4\omega^2\alpha^2$

résonance $\Leftrightarrow$ $f(\omega)$ admet 1 minimum

$\frac{df}{d\omega} = 0 \Leftrightarrow -4\omega(\omega_0^2 - \omega^2) + 8\omega\alpha^2 = 0$ $\omega \neq 0$!

$\Leftrightarrow \omega^2 = \omega_0^2 - 2\alpha^2$ $\omega = \sqrt{\omega_0^2 - 2\alpha^2}$

$\exists$ si $\omega_0 > \alpha\sqrt{2}$

5) analogie : résonance en tension aux bornes du condensateur d'1 circuit RLC série

Chimie

1) $v = k [BrO_3^-]^a [Br^-]^b [H_3O^+]^c$

2) large excès de Br^- et H_3O^+ $\Rightarrow$ $[Br^-] \simeq Cte$, $[H_3O^+] \simeq Cte$

dégénérescence de l'ordre $\Rightarrow$ $v = k_{app} [BrO_3^-]^a$ $k_{app} = k [Br^-]_0 [H_3O^+]_0$

3) à $t_{1/2}$: $[BrO_3^-](t_{1/2}) = \frac{[BrO_3^-]_0}{2}$ $t_{1/2} \simeq 2$ ms

4) cf cours

5) si $a = 1$ $\ln [BrO_3^-](t)$ est 1 droite
si $a = 2$ $\frac{1}{[BrO_3^-]}(t)$ est 1 droite d'après la figure 4 : $a = 1$.

6) si on double $[H_3O^+]_0$, v_0 est multiplié par 4 $\Rightarrow$ $c = 2$
" multiplie $[Br^-]_0$ par 1,5, v_0 l'est aussi $\Rightarrow$ $b = 1$

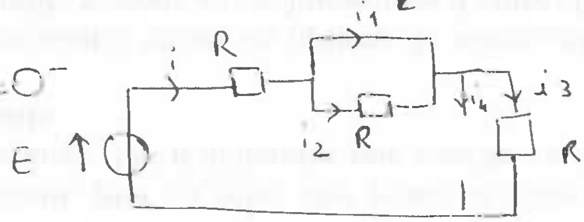
7) $v = k [BrO_3^-][H_3O^+]^2[Br^-]$

$k = 41$ $L^3 \cdot mol^{-3} \cdot s^{-1}$

Elec

bobine = fil en régime permanent

1) à $t = 0^-$

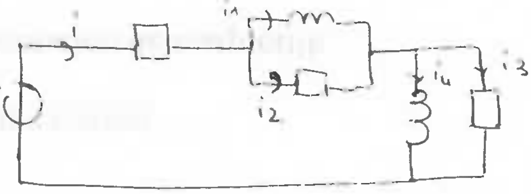


$i_4(0^-) = 0$ $\quad$ $i_1(0^-) = \frac{E}{2R}$

continuité

$\boxed{i_4(0^+) = 0}$ $\quad$ $\boxed{i_1(0^+) = \frac{E}{2R}}$

à $t = 0^+$



$i = i_1 + i_2 = i_3 + i_4$ $\quad$ à 0^+ : $i(0^+) = i_3(0^+) = \frac{E}{2R} + i_2(0^+)$

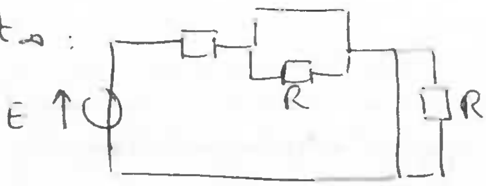
$E = Ri + Ri_3 + Ri_2 \iff E = Ri(0^+) + Ri(0^+) + R\left(i(0^+) - \frac{E}{2R}\right)$

$s = Ri_3$

$\boxed{i(0^+) = \frac{E}{2R} = i_3(0^+)}$ $\quad$ $\boxed{i_2(0^+) = 0}$

$\boxed{s(0^+) = \frac{E}{2}}$

2) à t_∞ :



$i_\infty = i_{1\infty} = i_{4\infty} = \frac{E}{R}$

$i_{2\infty} = 0$ et $i_{3\infty} = 0$ (courts-circuits !)

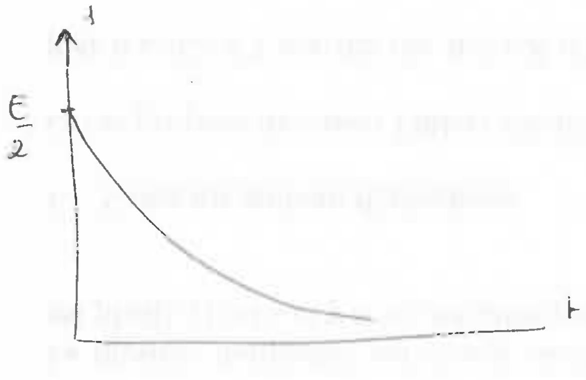
$\hookrightarrow s_\infty = 0$

3) $\omega_0^2 = \frac{1}{3\tau^2}$ $\quad$ $\omega_0 = \frac{R}{\sqrt{3}\,L}$ $\quad$ $\frac{\omega_0}{Q} = \frac{4}{3\tau}$ $\quad$ $Q = \frac{\sqrt{3}}{4} < 1/2 \Rightarrow$ régime apériodique $(\Rightarrow \Delta > 0)$

$s(t) = A\exp(r_1 t) + B\exp(r_2 t)$

$$\begin{cases} 3r^2 + \frac{4}{\tau} r + \frac{1}{\tau^2} = 0 & \Delta = \frac{4}{\tau^2} \\ r_1 = -\frac{1}{3\tau} & r_2 = -\frac{1}{\tau} \end{cases}$$

$s(t) = A\exp\left(-\frac{t}{3\tau}\right) + B\exp(-t/\tau)$



r_1 et $r_2 < 0$ $s(t)$ n'admet pas d'extremum