DM4

Exercice 1

1) $v_1 = -\dfrac{d[CH_3CH_2OH]_{estomac}}{dt}$

$CH_3CH_2OH_{estomac} \longrightarrow CH_3CH_2OH_{sang}$

$t=0$ $\quad C_1V_1 \quad\quad 0$

t $\quad (C_1-x)V_1 \quad\quad xV_1$

$v_1 = \dfrac{dx}{dt}$

2) on trace $\ln[CH_3CH_2OH]_{estomac}(t)$, on fait une régression linéaire :

$r^2 \simeq 1 \rightarrow$ ordre 1 validé

pente $= -k_1 \rightarrow k_1 = 0{,}17\ min^{-1}$

3) a) b) $[CH_3CH_2OH]_{sang} = \dfrac{xV_1}{V_2}$ $\quad xV_1 = n(CH_3CH_2OH_{sang})$ à t

c) $C_2 = \dfrac{xV_1}{V_2}$

à $t = 18$ min : $[CH_3CH_2OH]_{estomac} = 0{,}2$ mol/L $(= C_1 - x)$

à $t = 0$ $\quad [CH_3CH_2OH]_{estomac} = 4$ mol/L $(= C_1)$

$C_2 = \dfrac{(4-0{,}2)\times 250\times 10^{-3}}{40} = 0{,}024$ mol/L

d) $v = \dfrac{d[CH_3CH_2OH]_{sang}}{dt} = \dfrac{dx}{dt}\times\dfrac{V_1}{V_2} \rightarrow v = \dfrac{V_1}{V_2}v_1$

4) $v_2 = -\dfrac{d[CH_3CH_2OH]_{sang}}{dt}$

5) on trace $[CH_3CH_2OH]_{sang}(t)$, on fait une régression linéaire :

$r^2 \simeq 1 \rightarrow$ ordre 0 validé

pente $= -k_2 \rightarrow k_2 = 7{,}1\times 10^{-5}\ mol.L^{-1}.min^{-1}$

6) $C_{max} = \dfrac{0{,}5}{46} = 0{,}011$ mol/L

7) $\dfrac{dC}{dt} = \underset{\substack{\uparrow \\ \text{apparition}}}{v} - \underset{\substack{\uparrow \\ \text{disparition}}}{v_2} = \dfrac{V_1}{V_2}\dfrac{dx}{dt} - v_2$

ordre 1 $\Rightarrow v_1 = \dfrac{dx}{dt} = k_1[CH_3CH_2OH]_{estomac}$
$= k_1C_1\exp(-k_1t)$

ordre 0 $\Rightarrow v_2 = k_2$

$$\Rightarrow \frac{dC_2}{dt} = \frac{V_1}{V_2} k_1 C_1 \exp(-k_1 t) - k_2$$

8) $C_2 = -\frac{V_1}{V_2} C_1 \exp(-k_1 t) - k_2 t + Cte$

or à $t=0$ $C_2 = 0$

donc $Cte = \frac{V_1}{V_2} C_1$

$\rightarrow C_2 = \frac{V_1}{V_2} C_1 (1 - \exp(-k_1 t)) - k_2 t$

9) $V_1 = 66\ cL = 0,66L$

$C_1 = \frac{0,9}{0,66}$ } $C_1 V_1 = 0,9\ mol$ $\quad V_2 = 40L$

C_2 est maximale $\Leftrightarrow \left(\frac{dC_2}{dt}\right) = 0 \Leftrightarrow \frac{V_1}{V_2} k_1 C_1 \exp(-k_1 t_{max}) = k_2$

$$t_{max} = \ln\left(\frac{k_2 V_2}{k_1 C_1 V_1}\right) \times \frac{1}{-k_1} = 23,5\ min$$

10) $C_2(t_{max}) = 0,02\ mol/L > C_{max}$ $\rightarrow$ il ne peut pas conduire

11) si $t \rightarrow \infty$ $\quad C_2(t) \simeq -k_2 t + \frac{V_1}{V_2} C_1$ $\Leftrightarrow$ droite de pente $(-k_2)$

$C_2 = C_{max} \Leftrightarrow -k_2 t_{ok} + \frac{V_1}{V_2} C_1 = 0,011 \Rightarrow t_{ok} = 162$ min soit 2h42min

Exercice 2

1) $ml\ddot{\theta} + \alpha l \dot{\theta} + mg\theta = 0 \Leftrightarrow$ oscillateur harmonique amorti

2) $\ddot{\theta} + \frac{\alpha}{m}\dot{\theta} + \frac{g}{l}\theta = 0 \qquad \omega_0^2 = \frac{g}{l} \Rightarrow \omega_0 = \sqrt{\frac{g}{l}}$

$\frac{\omega_0}{Q} = \frac{\alpha}{m} \rightarrow Q = \frac{\omega_0 m}{\alpha} = \sqrt{\frac{g}{l}} \times \frac{m}{\alpha}$

3) a) le régime est pseudo périodique $\Leftrightarrow Q > \frac{1}{2}$

b) $X^2 + \frac{\omega_0}{Q} X + \omega_0^2 = 0 \qquad \Delta = \omega_0^2\left(\frac{1}{Q^2} - 4\right) = 4\omega_0^2\left(\frac{1}{4Q^2} - 1\right)$

$X_{1,2} = \frac{-\frac{\omega_0}{Q} \pm i\sqrt{4\omega_0^2\left(1 - \frac{1}{4Q^2}\right)}}{2}$ • pseudo-pulsation $\Omega = \omega_0\sqrt{1 - \frac{1}{4Q^2}}$

• pseudo-période $T = \frac{2\pi}{\Omega} = \frac{2\pi}{\omega_0}\left(1 - \frac{1}{4Q^2}\right)^{-1/2}$

on mesure $T = 2,20 - 1,10 = 1,10\ s$

c) $\theta(t) = \exp\left(-\frac{\omega_0 t}{2Q}\right)\left(A\cos(\Omega t) + B\sin(\Omega t)\right)$

$\theta(0) = \theta_0 \rightarrow A = \theta_0$

$\dot{\theta}(0) = 0 \rightarrow \frac{-\omega_0}{2Q} \times A + B\Omega = 0 \qquad B = \frac{\omega_0 \theta_0}{2Q\,\Omega}$

$\rightarrow \theta(t) = \theta_0 \exp\left(-\frac{\omega_0 t}{2Q}\right)\left(\cos(\Omega t) + \frac{\omega_0}{2Q\Omega}\sin(\Omega t)\right)$

d) $$\frac{\theta(t)}{\theta(t+T)} = \frac{\exp\left(-\frac{\omega_0 t}{2Q}\right)\theta_0\left(\cos(\Omega t) + \frac{\omega_0}{2Q\Omega}\sin(\Omega t)\right)}{\exp\left(-\frac{\omega_0}{2Q}(t+T)\right)\theta_0\left(\cos(\Omega(t+T)) + \frac{\omega_0}{2Q\Omega}\sin(\Omega(t+T))\right)} = \exp\left(\frac{\omega_0 T}{2Q}\right)$$

$$\left(\begin{array}{l}\cos(\Omega t) = \cos(\Omega(t+T)) \\ \sin(\Omega t) = \sin(\Omega(t+T))\end{array}\right)$$

$\hookrightarrow \delta = \frac{\omega_0 T}{2Q} = \ln\left(\frac{8,95}{8,02}\right) = 0,110$

e) $\frac{\omega_0 T}{2Q} = \delta \iff \frac{\omega_0}{Q} = \frac{2 \times 0,11}{1,10} = 0,20\,s^{-1}$

or $\alpha = \frac{\omega_0 m}{Q} = 9,4 \times 10^{-2}\ kg \cdot s^{-1}$

$\Omega^2 = \omega_0^2 - \frac{\omega_0^2}{4Q^2} \iff \omega_0^2 = \Omega^2 + \frac{\omega_0^2}{4Q^2} = \left(\frac{2\pi}{T}\right)^2 + \frac{1}{4}\left(\frac{\omega_0}{Q}\right)^2$

$\omega_0 = \sqrt{\left(\frac{2\pi}{1,1}\right)^2 + \frac{1}{4} \times 0,2^2} = 5,71\ rad \cdot s^{-1}$

$Q = \frac{\omega_0 T}{2\delta} = \frac{5,71 \times 1,10}{2 \times 0,110} = 28,6$