Réponses

1) $\ln 2$.

2) Si $x < -1$ alors $F(x) = \ln(-1-x)$ d'où $F'(x) = \dfrac{-1}{-1-x} = f(x)$.

3) $x \mapsto -\ln(\cos x)$.

4) $F(x) = \dfrac{1}{2}(\sin x - \cos x)e^x$.

5) $F(x) = \dfrac{1}{10}(5 - \cos 2x - 2\sin 2x)e^x$.

6) $F(x) = \arctan x + \dfrac{1}{2}\ln(1+x^2)$.

7) $F(x) = \dfrac{1}{4}\ln(1+x^2) + \dfrac{1}{2}\arctan x + \dfrac{1}{2}\ln|1+x|$.

8) $\sqrt{3} - \dfrac{\pi}{3}$.

9) $\dfrac{11}{6}$.

10) $\dfrac{16}{15}$.

11) $\dfrac{\pi}{4} - \dfrac{1}{2}\ln 2$.

12) $\dfrac{1}{2}$, en remarquant que $t^3e^{t^2} = \dfrac{1}{2}t^2 \times 2te^{t^2}$.

13) $x \mapsto x\ \arctan x - \dfrac{1}{2}\ln(1+x^2)$.

14) $\dfrac{e^2-3}{4}$ en remarquant que $I_n = \displaystyle\int_1^e t(\ln t)^n\ \mathrm{d}t$ vérifie la relation de récurrence $I_n = \dfrac{1}{2}(e^2 - nI_{n-1})$.

15) $\dfrac{2}{15}$.

16) 1.

17) En posant $x = \frac{\pi}{4} - t$ on obtient $I = \dfrac{1}{4}\pi\ln 2 - I$ d'où $I = \dfrac{1}{8}\pi\ln 2$.

18) On remarque que $\ln(1+x) - x + \dfrac{x^2}{2} = \displaystyle\int_0^x \dfrac{(x-t)^2}{2}\ln^{(3)}(1+t)\ \mathrm{d}t \geqslant 0$ et que $\ln(1+x) - x + \dfrac{x^2}{2} - \dfrac{x^3}{3} = \displaystyle\int_0^x \dfrac{(x-t)^3}{6}\ln^{(4)}(1+t)\ \mathrm{d}t \leqslant 0$.

19) $\dfrac{1}{\sqrt{1-x}} \underset{0}{=} 1 + \displaystyle\sum_{k=1}^{k=n} \dfrac{1\times 3\times 5\times\cdots\times(2k-1)}{k!\ 2^k}x^k\ + o(x^n)$.

20) $f(x) \underset{0}{\sim} \dfrac{x^5}{60}$.

21) $\displaystyle\lim_{\substack{x\to 0\\ x>0}} f(x) = \lim_{\substack{x\to 0\\ x<0}} f(x) = -1$.

22) $2(x+1) - 2(x+1)^2 + (x+1)^3$ si $n \geqslant 3$.

23) $\dfrac{2}{3} + \dfrac{1}{3}(x-1) + \dfrac{4}{9}(x-1)^2 - \dfrac{2}{9}(x-1)^3 + o((x-1)^3)$.

24) $\displaystyle\lim_{\substack{x\to 1\\ x>1}} f(x) = \lim_{\substack{x\to 1\\ x<1}} f(x) = -2$.

25) On montre que $f(x) = \dfrac{1}{X} + 1 + \dfrac{1}{2}X + o(X) = x + 1 + \dfrac{1}{2x} + o\left(\dfrac{1}{x}\right)$.

26) $\dfrac{1}{\sin x} - \dfrac{1}{x} \underset{0}{=} \dfrac{1}{6}x + o(x)$ donc $f(0) = 0$ et $f'(0) = \dfrac{1}{6}$.

27) $\sqrt{1+\sin x} \underset{0}{=} 1 + \dfrac{1}{2}x - \dfrac{1}{8}x^2 - \dfrac{1}{48}x^3 + o(x^3)$.

28) $\arccos x \underset{0}{=} \dfrac{\pi}{2} - x - \dfrac{x^3}{6} - \dfrac{3x^5}{40} + o(x^6)$.