

(1) - لنثبت أن ABC مثلث قائم الزاوية.

$$\left. \begin{array}{l} AB^2 = 6^2 = 36 \\ AC^2 = 8^2 = 64 \\ BC^2 = 10^2 = 100 \end{array} \right\} \text{ لدينا : } \text{ إذن ، } BC^2 = AB^2 + AC^2 .$$

وحسب مبرهنة فيثاغورس مباشرة فإن ABC مثلث قائم الزاوية في A .

(2) - لنحسب النسب المثلثية للزاوية $\hat{A}BC$:

* / لدينا ABC مثلث قائم الزاوية في A .

$$\boxed{\cos \hat{A}BC = \frac{3}{5}}$$

$$\boxed{\sin \hat{A}BC = \frac{4}{5}}$$

$$\boxed{\tan \hat{A}BC = \frac{4}{3}}$$

$$\cos \hat{A}BC = \frac{6}{10}$$

$$\sin \hat{A}BC = \frac{8}{10}$$

$$\tan \hat{A}BC = \frac{8}{6}$$

$$\cos \hat{A}BC = \frac{AB}{BC}$$

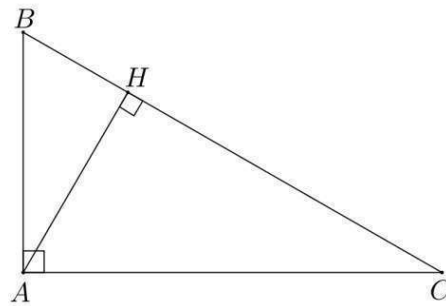
$$\sin \hat{A}BC = \frac{AC}{BC}$$

$$\tan \hat{A}BC = \frac{AC}{AB}$$

إذن : و

، أي : و

إذن : و



(3) - الشكل :

(4) - / * لنحسب : AH .

بما أن H إسقاط العمودي للنقطة A على المستقيم (BC) ، فإن ABH مثلث قائم الزاوية في H .

$$\sin \hat{ABH} = \frac{AH}{6} \quad \text{،} \quad \sin \hat{ABH} = \frac{AH}{AB}$$

و بما أن $\hat{ABH} = \hat{ABC}$ (نفس الزاوية) ، فإن $\sin \hat{ABH} = \sin \hat{ABC}$:

$$AH = \frac{6 \times 4}{5} \quad \text{يعني أن} \quad \frac{AH}{6} = \frac{4}{5}$$

$$\boxed{AH = \frac{24}{5} \text{ cm}} \quad \text{و بالتالي فإن}$$

/ * لنحسب : CH .

بما أن H إسقاط العمودي للنقطة A على المستقيم (BC) ، فإن ACH مثلث قائم الزاوية في H .

$$8^2 = \left(\frac{24}{5}\right)^2 + CH^2 \quad \text{،} \quad AC^2 = AH^2 + CH^2 \quad \text{إذن حسب مبرهنة فيثاغورس مباشرة فإن}$$

و منه فإن :

$$CH^2 = 8^2 - \left(\frac{24}{5}\right)^2 = 64 - \frac{576}{25} = \frac{1600 - 576}{25} = \frac{1024}{25}$$

$$\boxed{CH = \frac{32}{5} \text{ cm}} \quad \text{و بالتالي فإن} \quad CH > 0 \quad \text{فإن} \quad CH = \sqrt{\frac{1024}{25}}$$

$$\begin{aligned} C &= \cos^4 \alpha - \sin^4 \alpha - \cos^2 \alpha + 3 \sin^2 \alpha \\ &= (\cos^2 \alpha)^2 - (\sin^2 \alpha)^2 - \cos^2 \alpha + 3 \sin^2 \alpha \\ &= (\cos^2 \alpha + \sin^2 \alpha)(\cos^2 \alpha - \sin^2 \alpha) - \cos^2 \alpha + 3 \sin^2 \alpha \\ &= 1 \times (\cos^2 \alpha - \sin^2 \alpha) - \cos^2 \alpha + 3 \sin^2 \alpha \\ &= \cos^2 \alpha - \sin^2 \alpha - \cos^2 \alpha + 3 \sin^2 \alpha \\ &= 2 \sin^2 \alpha \end{aligned}$$

$$\begin{aligned} D &= \sin \alpha \times \sqrt{1 - \cos \alpha} \times \sqrt{1 + \cos \alpha} + \cos^2 \alpha \\ &= \sin \alpha \times \sqrt{(1 - \cos \alpha)(1 + \cos \alpha)} + \cos^2 \alpha \\ &= \sin \alpha \times \sqrt{1^2 - \cos^2 \alpha} + \cos^2 \alpha \\ &= \sin \alpha \times \sqrt{1 - \cos^2 \alpha} + \cos^2 \alpha \\ &= \sin \alpha \times \sqrt{\sin^2 \alpha} + \cos^2 \alpha \\ &= \sin \alpha \times \sin \alpha + \cos^2 \alpha \\ &= \sin^2 \alpha + \cos^2 \alpha \\ &= 1 \end{aligned}$$

(1) - لنسب ما يلي :

$$\begin{aligned} A &= \cos \alpha (\sin \alpha + \cos \alpha) - \sin \alpha (\cos \alpha - \sin \alpha) \\ &= \cos \alpha \times \sin \alpha + \cos^2 \alpha - \sin \alpha \times \cos \alpha + \sin^2 \alpha \\ &= \cos \alpha \times \sin \alpha - \cos \alpha \times \sin \alpha + \cos^2 \alpha + \sin^2 \alpha \\ &= 0 + 1 \\ &= 1 \end{aligned}$$

$$\begin{aligned} B &= \frac{1}{1 + \sin \alpha} + \frac{1}{1 - \sin \alpha} - \frac{2}{\cos^2 \alpha} \\ &= \frac{(1 - \sin \alpha) + (1 + \sin \alpha)}{(1 - \sin \alpha)(1 + \sin \alpha)} - \frac{2}{\cos^2 \alpha} \\ &= \frac{1 - \sin \alpha + 1 + \sin \alpha}{1^2 - \sin^2 \alpha} - \frac{2}{\cos^2 \alpha} \\ &= \frac{2}{1 - \sin^2 \alpha} - \frac{2}{\cos^2 \alpha} \\ &= \frac{2}{\cos^2 \alpha} - \frac{2}{\cos^2 \alpha} \\ &= 0 \end{aligned}$$

$$\sqrt{1-\sin \alpha} \times \sqrt{1+\cos \alpha} = \cos \alpha : \text{لنبين أن}^*$$

لدينا :

$$\begin{aligned} \sqrt{1-\sin \alpha} \times \sqrt{1+\cos \alpha} &= \sqrt{(1-\sin \alpha)(1+\cos \alpha)} \\ &= \sqrt{1^2 - \sin^2 \alpha} \\ &= \sqrt{1-\sin^2 \alpha} \\ &= \sqrt{\cos^2 \alpha} \\ &= \cos \alpha \end{aligned}$$

$$\boxed{\sqrt{1-\sin \alpha} \times \sqrt{1+\cos \alpha} = \cos \alpha} : \text{إذن}$$

$$\sin^2 \alpha = \frac{\tan^2}{1+\tan^2} : \text{لنبين أن}^*$$

لدينا :

$$\begin{aligned} \frac{\tan^2 \alpha}{1+\tan^2 \alpha} &= \frac{\frac{\sin^2 \alpha}{\cos^2 \alpha}}{\frac{1}{\cos^2 \alpha}} \\ &= \frac{\sin^2 \alpha}{\cos^2 \alpha} \times \frac{\cos^2 \alpha}{1} \\ &= \sin^2 \alpha \end{aligned}$$

$$\boxed{\sin^2 \alpha = \frac{\tan^2}{1+\tan^2}} : \text{إذن}$$

$$1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha} : \text{لنبين أن}^*$$

لدينا :

$$\begin{aligned} 1 + \tan^2 \alpha &= 1 + \frac{\sin^2 \alpha}{\cos^2 \alpha} \\ &= \frac{\cos^2 \alpha}{\cos^2 \alpha} + \frac{\sin^2 \alpha}{\cos^2 \alpha} \\ &= \frac{\cos^2 \alpha + \sin^2 \alpha}{\cos^2 \alpha} \\ &= \frac{1}{\cos^2 \alpha} \end{aligned}$$

$$\boxed{1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha}} : \text{إذن}$$

$$\begin{aligned} E &= (\cos \alpha + \sin \alpha)^2 + (\cos \alpha - \sin \alpha)^2 \\ &= \cos^2 \alpha + 2 \cos \alpha \times \sin \alpha + \sin^2 \alpha + \cos^2 \alpha - 2 \cos \alpha \times \sin \alpha + \sin^2 \alpha \\ &= 2 \cos^2 \alpha + 2 \sin^2 \alpha \\ &= 2(\cos^2 \alpha + \sin^2 \alpha) \\ &= 2 \times 1 \\ &= 2 \end{aligned}$$

$$\begin{aligned} F &= \sqrt{2} \times \sin^2 \alpha + 2 \sin 45^\circ \times \cos^2 \alpha \\ &= \sqrt{2} \times \sin^2 \alpha + 2 \times \frac{\sqrt{2}}{2} \times \cos^2 \alpha \\ &= \sqrt{2} \times \sin^2 \alpha + \sqrt{2} \times \cos^2 \alpha \\ &= \sqrt{2}(\sin^2 \alpha + \cos^2 \alpha) \\ &= \sqrt{2} \times 1 \\ &= \sqrt{2} \end{aligned}$$

$$\frac{\cos^4 \alpha - \sin^4 \alpha}{\cos^2 \alpha - \sin^2 \alpha} = 1 : \text{لنبين أن}^* - (2)$$

لدينا :

$$\begin{aligned} \frac{\cos^4 \alpha - \sin^4 \alpha}{\cos^2 \alpha - \sin^2 \alpha} &= \frac{(\cos^2 \alpha)^2 - (\sin^2 \alpha)^2}{\cos^2 \alpha - \sin^2 \alpha} \\ &= \frac{(\cos^2 \alpha + \sin^2 \alpha)(\cos^2 \alpha - \sin^2 \alpha)}{\cos^2 \alpha - \sin^2 \alpha} \\ &= \frac{1 \times (\cos^2 \alpha - \sin^2 \alpha)}{\cos^2 \alpha - \sin^2 \alpha} \\ &= \frac{\cos^2 \alpha - \sin^2 \alpha}{\cos^2 \alpha - \sin^2 \alpha} \\ &= 1 \end{aligned}$$

$$\frac{1-\cos \alpha}{\sin \alpha} = \frac{\sin \alpha}{1+\cos \alpha} : \text{لنبين أن}^*$$

$$\left. \begin{aligned} (1-\cos \alpha)(1+\cos \alpha) &= 1-\cos^2 \alpha = \sin^2 \alpha \\ \sin \alpha \times \sin \alpha &= \sin^2 \alpha \end{aligned} \right\} \text{لدينا : 9}$$

$$(1-\cos \alpha)(1+\cos \alpha) = \sin \alpha \times \sin \alpha : \text{إذن}$$

$$\frac{1-\cos \alpha}{\sin \alpha} = \frac{\sin \alpha}{1+\cos \alpha} : \text{9 بالتالي فإن}$$

(3) -- / * حساب $\cos \alpha$:

لدينا : $\sin^2 \alpha + \cos^2 \alpha = 1$ يعني أن :

$$\cos^2 \alpha = 1 - \sin^2 \alpha$$

$$\cos^2 \alpha = 1 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$\cos^2 \alpha = 1 - \frac{3}{4}$$

$$\cos^2 \alpha = \frac{4-3}{4}$$

$$\cos^2 \alpha = \frac{1}{4}$$

و بما أن : $0 < \cos \alpha < 1$ فإن : $\cos \alpha = \sqrt{\frac{1}{4}}$

$$\boxed{\cos \alpha = \frac{1}{2}} \quad \text{أى} :$$

/ * حساب $\tan \alpha$:

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha} \quad \text{نعلم أن}$$

$$\tan \alpha = \frac{\sqrt{3}}{\frac{1}{2}} = \frac{2\sqrt{3}}{1} = 2\sqrt{3} \quad \text{و منه فإن} \quad \tan \alpha = \frac{2}{1} \quad \text{و بالتالي} \quad \boxed{\tan \alpha = \sqrt{3}}$$

(ب) -- / * حساب :

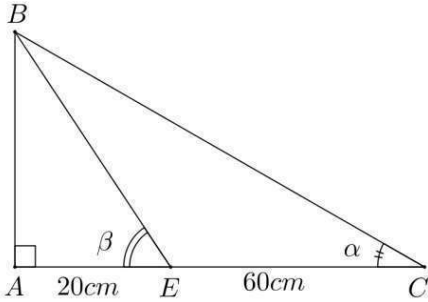
$$\sin(90^\circ - \alpha) \quad \text{و} \quad \cos(90^\circ - \alpha) \quad \text{و} \quad \tan(90^\circ - \alpha)$$

$$(90^\circ - \alpha) + \alpha = 90^\circ + \alpha - \alpha = 90^\circ \quad \text{لدينا} :$$

$$\sin(90^\circ - \alpha) = \cos \alpha = \frac{1}{2} \quad \text{إذن} :$$

$$\cos(90^\circ - \alpha) = \sin \alpha = \frac{\sqrt{3}}{2}$$

$$\tan(90^\circ - \alpha) = \frac{1}{\tan \alpha} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$



لنحسب : AB

/ * لدينا من خلال الشكل :

ABC مثلث قائم الزاوية في A و ABE مثلث قائم الزاوية في A .

$$\left. \begin{aligned} \tan \hat{ACB} &= \frac{AB}{AC} \\ \tan \hat{AEB} &= \frac{AB}{AE} \end{aligned} \right\} \text{و} \quad \text{إذن} :$$

$$\left. \begin{aligned} \tan \alpha &= \frac{AB}{80} \\ \tan \beta &= \frac{AB}{20} \end{aligned} \right\} \text{و} \quad \text{أى} :$$

و بما أن : $\alpha + \beta = 90^\circ$ فإن : $\tan \alpha = \frac{1}{\tan \beta}$

$$\frac{AB}{80} = \frac{1}{\frac{AB}{20}} \quad \text{أى} :$$

و منه فإن : $\frac{AB}{80} = \frac{20}{AB}$ ، يعني أن : $AB^2 = 1600$

و بما أن : $AB > 0$ فإن : $AB = \sqrt{1600}$ ، و بالتالي فإن : $\boxed{AB = 40 \text{ cm}}$

(1) - / * حساب : $\cos \hat{A}BC$.

لدينا : $\cos^2 \hat{A}BC + \sin^2 \hat{A}BC = 1$ ، يعني أن $\cos^2 \hat{A}BC = 1 - \sin^2 \hat{A}BC$

$$\cos^2 \hat{A}BC = 1 - \left(\frac{3}{5}\right)^2 = 1 - \frac{9}{25} = \frac{25}{25} - \frac{9}{25} = \frac{16}{25} \quad \text{أي}$$

و بما أن : $0 < \cos \hat{A}BC < 1$ فإن : $\cos \hat{A}BC = \sqrt{\frac{16}{25}}$ ، و بالتالي فإن : $\cos \hat{A}BC = \frac{4}{5}$.

/ * حساب : $\tan \hat{A}BC$.

لدينا : $\tan \hat{A}BC = \frac{\sin \hat{A}BC}{\cos \hat{A}BC}$ ، أي : $\tan \hat{A}BC = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{5} \times \frac{5}{4} = \frac{3}{4}$ ، إذن : $\tan \hat{A}BC = \frac{3}{4}$.

(2) - حساب : AB و AC .

لدينا ABC مثلث قائم الزاوية في A .

$$\left. \begin{array}{l} \frac{4}{5} = \frac{AB}{15} \\ \frac{3}{5} = \frac{AC}{BC} \end{array} \right\} \text{ أي : ،} \quad \left. \begin{array}{l} \cos \hat{A}BC = \frac{AB}{BC} \\ \sin \hat{A}BC = \frac{AC}{BC} \end{array} \right\} \text{ إذن :}$$

$$\left. \begin{array}{l} AB = 12 \text{ cm} \\ AC = 9 \text{ cm} \end{array} \right\} \text{ و بالتالي فإن : ،} \quad \left. \begin{array}{l} AB = \frac{60}{5} \\ AC = \frac{45}{5} \end{array} \right\} \text{ يعني أن :}$$

(1) - لنحسب : $A = 2 \cos 15^\circ + \cos^2 36^\circ - 2 \sin 75^\circ + \cos^2 54^\circ$.

$$\left. \begin{array}{l} \cos 15^\circ = \sin 75^\circ \\ \cos 36^\circ = \cos 54^\circ \end{array} \right\} \text{ إذن : ،} \quad \left. \begin{array}{l} 15^\circ + 75^\circ = 90^\circ \\ 36^\circ + 54^\circ = 90^\circ \end{array} \right\} \text{ لدينا :}$$

و منه فإن :

$$\begin{aligned} A &= 2 \cos 15^\circ + \cos^2 36^\circ - 2 \sin 75^\circ + \cos^2 54^\circ \\ &= 2 \sin 75^\circ + \sin^2 54^\circ - 2 \sin 75^\circ + \cos^2 54^\circ \\ &= 2 \sin 75^\circ - 2 \sin 75^\circ + \sin^2 54^\circ + \cos^2 54^\circ \\ &= 0 + 1 \\ &= 1 \end{aligned}$$

$$(2) - \text{لنحسب} : B = \cos^2 28^\circ - \sin^2 51^\circ + \cos^2 62^\circ + \cos^2 39^\circ$$

$$\left. \begin{array}{l} \cos 28^\circ = \sin 62^\circ \\ \sin 51^\circ = \cos 39^\circ \end{array} \right\} \text{لدينا} : \text{إذن} , \left. \begin{array}{l} 28^\circ + 62^\circ = 90^\circ \\ 51^\circ + 39^\circ = 90^\circ \end{array} \right\} \text{و}$$

و منه فإن :

$$\begin{aligned} B &= \cos^2 28^\circ - \sin^2 51^\circ + \cos^2 62^\circ + \cos^2 39^\circ \\ &= \sin^2 62^\circ - \cos^2 39^\circ + \cos^2 62^\circ + \cos^2 39^\circ \\ &= \sin^2 62^\circ + \cos^2 62^\circ - \cos^2 39^\circ + \cos^2 39^\circ \\ &= 1 + 0 \\ &= 1 \end{aligned}$$

$$(3) - \text{لنحسب} : C = \tan 73^\circ \times \tan 17^\circ - \sin^2 40^\circ - \sin^2 50^\circ$$

$$\left. \begin{array}{l} \tan 73^\circ = \frac{1}{\tan 17^\circ} \\ \sin 40^\circ = \cos 50^\circ \end{array} \right\} \text{لدينا} : \text{إذن} , \left. \begin{array}{l} 73^\circ + 17^\circ = 90^\circ \\ 40^\circ + 50^\circ = 90^\circ \end{array} \right\} \text{و}$$

و منه فإن :

$$\begin{aligned} C &= \tan 73^\circ \times \tan 17^\circ - \sin^2 40^\circ - \sin^2 50^\circ \\ &= \frac{1}{\tan 17^\circ} \times \tan 17^\circ - \cos^2 50^\circ - \sin^2 50^\circ \\ &= \frac{\tan 17^\circ}{\tan 17^\circ} - (\cos^2 50^\circ + \sin^2 50^\circ) \\ &= 1 - 1 \\ &= 0 \end{aligned}$$